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A Fundamental Limit on Antenna Gain for Electrically Small Antennas

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***Abstract*—A fundamental limit on an antenna's gain is derived and compared to measurements taken on a number of different antennas. First, a propagation formula applicable in both the near and far fields is developed, and that result is used to demonstrate that the gain of an antenna is limited by its electrical size.**

***Index Terms*—Electrically Small Antennas, Antenna Gain, Antenna Measurements, Near Field.**

I. Introduction

THE area around antennas is often split into two areas: the near field, which extends out from the antenna at a distance comparable to a wavelength, and the far field, which is the area beyond the near field. See [1] for an excellent discussion of the properties of the near field, the far field, and exactly where the boundary between the two lies. For most antenna applications, especially those at high frequencies, the behavior of the antenna's electromagnetic fields in the near field is often of little consequence. For example, a typical Wi-Fi[1] signal operating at 2.4 GHz has a wavelength of about 12.5 cm. Because most Wi-Fi applications operate at ranges on the order of meters, this system lends itself well to far-field analysis. This is the case for almost every electromagnetic system in common use today.

However, the near field has some interesting properties that some systems exploit. As we will demonstrate, the power transmitted by a near-field link rolls off much faster than in the far field, which means that often times signals will not transmit far enough to cause harmful interference. In turn, short distance, high data-rate communication is possible using a near-field communication link.

Another novel use of the near field is in real-time location systems (RTLS). The Q-Track Corporation has pioneered a RTLS technology known as Near-Field Electromagnetic Ranging (NFER)[2] that takes advantage of the fact that the phase difference of a transmitting antenna's electric and magnetic fields goes to zero as the distance from the antenna increases [2].

Often near-field applications like these use low frequencies in order to fully realize the advantages of the near field at substantial distances. Compare the wavelength of a typical Wi-Fi signal (12.5 cm) with the wavelength of a typical Q-Track NFER signal (1 MHz, 300 m). As a consequence, most antennas used in near-field systems are much smaller than a wavelength.

This paper exploits near-field phenomena in order to derive a fundamental limit on the gain of an antenna versus its electrical size. First, we derive a propagation formula similar to Friis's formula for the far field. Using this formula, we show a fundamental limit on the gain of an antenna versus its electrical size, and we compare the limit to a number of gain measurements taken on actual antennas.

II. Friis's Propagation Formula and the Far Field

Harald Friis derived the propagation formula that bears his name in the following form [3]

$$\frac{P_{RX}}{P_{TX}} = \frac{A_{RX} A_{TX}}{(d\lambda)^2} . \tag{1}$$

P_{RX} is the power received by the receiving antenna, P_{TX} is the power transmitted by the transmitting antenna, A_{RX} is the effective area of the receive antenna, A_{TX} is the effective area of the transmit antenna, d is the distance between each antenna, and λ is the wavelength of the electromagnetic wave. Note that the antenna effective area or aperture A is related to the antenna gain G by

$$A = \frac{\lambda^2 G}{4\pi} , \tag{2}$$

so Friis's formula can also be written as

$$\frac{P_{RX}}{P_{TX}} = \left(\frac{\lambda}{4\pi d}\right)^2 G_{RX} G_{TX} = \frac{G_{RX} A_{TX}}{4\pi d^2} = \frac{A_{RX} G_{TX}}{4\pi d^2} . \tag{3}$$

This is a very powerful formula, but because it assumes a plane wave front, it is only applicable in the far field. In his paper, Friis warns that his formula "is correct to within a few percent when

$$d \geq \frac{2a^2}{\lambda} , \tag{4}$$

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[1] Wi-Fi is a registered trademark of the Wi-Fi Alliance.

[2] NFER is a registered trademark of the Q-Track Corporation.

where a is the largest linear dimension of either of the antennas" [3]. Specifically, "[t]his criterion has a phase error of one-sixteenth of a wavelength" [1]. Also, Friis's formula is only strictly valid in free space.

III. Near-Field Propagation Formula

Whereas the far-field propagation formula developed by Friis is the same regardless of the type of antenna used, in the near field one must distinguish between an electric antenna (like a dipole or whip) and its dual: a magnetic antenna (such as a loop). First the case of an electric transmit antenna is considered, followed by the case of a magnetic antenna.

A. *Electric Transmit Antenna*

Imagine an infinitesimal current element with length $\Delta l << \lambda$ in free space. Suppose the current element has a uniform current distribution I across its length. This structure, first introduced by Heinrich Hertz [4], has since been analyzed by a number of authors [5-7]. A theoretical idealization, it is an accurate model for the electrically small antennas being considered.

The time harmonic electric field at the point (r, θ, ϕ) generated by the infinitesimal dipole is given by

$$\vec{E} = \frac{I\Delta l k^2}{4\pi\varepsilon_0 c}\left(\left(\frac{j}{kr}+\frac{1}{(kr)^2}-\frac{j}{(kr)^3}\right)\sin(\theta)\hat{\theta} + 2\left(\frac{1}{(kr)^2}-\frac{j}{(kr)^3}\right)\cos(\theta)\hat{r}\right)e^{-jkr}, \quad (5)$$

where $k = 2\pi / \lambda$ is the wave number, ε_0 is the electric permittivity of free space, and c is the speed of light. The magnetic field is

$$\vec{H} = \frac{I\Delta l k^2}{4\pi}\left(\left(\frac{j}{kr}+\frac{1}{(kr)^2}\right)\sin(\theta)\hat{\phi}\right)e^{-jkr}. \quad (6)$$

The magnitudes squared of the fields are

$$\left|\vec{E}\right|^2 = \left(\frac{I\Delta l k^2}{4\pi\varepsilon_0 c}\right)^2\left(\left(\frac{1}{(kr)^2}-\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right)\sin^2(\theta) + 4\left(\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right)\cos^2(\theta)\right), \text{ and} \quad (7)$$

$$\left|\vec{H}\right|^2 = \left(\frac{I\Delta l k^2}{4\pi}\right)^2\left(\frac{1}{(kr)^2}+\frac{1}{(kr)^4}\right)\sin^2(\theta). \quad (8)$$

For the sake of compactness and for reasons that will be clearer momentarily, define the path-loss functions PL_{like} and PL_{unlike} as

$$PL_{like}(r,\theta,\phi) = \frac{3k^2}{8\pi}\left(\left(\frac{1}{(kr)^2}-\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right)\sin^2(\theta) + 4\left(\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right)\cos^2(\theta)\right), \text{ and} \quad (9)$$

$$PL_{unlike}(r,\theta,\phi) = \frac{3k^2}{8\pi}\left(\frac{1}{(kr)^2}+\frac{1}{(kr)^4}\right)\sin^2(\theta). \quad (10)$$

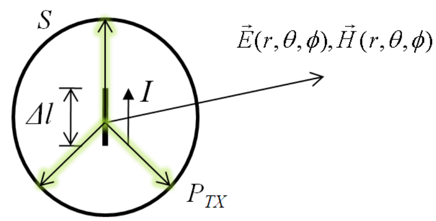

Fig. 1. Infinitesimal dipole transmitting with fields at (r, θ, ϕ).

which means that the electric and magnetic fields can be rewritten as

$$\left|\vec{E}\right|^2 = \frac{1}{6\pi}\left(\frac{I\Delta l k}{\varepsilon_0 c}\right)^2 PL_{like}, \text{ and} \left|\vec{H}\right|^2 = \frac{1}{6\pi}(I\Delta l k)^2 PL_{unlike}. \quad (11)$$

Define the power densities of the electric and magnetic fields, respectively as W_E and W_H,

$$W_E = \frac{1}{2}\varepsilon_0 c\left|\vec{E}\right|^2 = \frac{(I\Delta l k)^2}{12\pi\varepsilon_0 c}PL_{like}, \text{ and} \quad (12)$$

$$W_H = \frac{1}{2}\mu_0 c\left|\vec{H}\right|^2 = \frac{\mu_0 c(I\Delta l k)^2}{12\pi}PL_{unlike}. \quad (13)$$

where μ_0 is the magnetic permeability of free space.

Because the power transmitted by an ideal dipole through a closed sphere around the antenna as pictured in Fig. 1 is given by

$$P_{TX,E} = \frac{1}{2}\oiint_S \vec{E}\times\vec{H}^*\cdot d\vec{S} = \frac{(I\Delta l k)^2}{12\pi\varepsilon_0 c}, \quad (14)$$

the power densities (12) and (13) can be rewritten as

$$W_E = \frac{(I\Delta l k)^2}{12\pi\varepsilon_0 c}PL_{like} = P_{TX,E}PL_{like}, \text{ and} \quad (15)$$

$$W_H = \frac{\mu_0 c(I\Delta l k)^2}{12\pi}PL_{unlike} = c^2\mu_0\varepsilon_0 P_{TX,E}PL_{unlike} = P_{TX,E}PL_{unlike}. \quad (16)$$

The power density at the receive antenna is equal to the power it receives divided by its effective area, so

$$W_E = \frac{P_{RX,E}}{A_{nf,RX}} = P_{TX,E}PL_{like} \Rightarrow \frac{P_{RX,E}}{P_{TX,E}} = A_{nf,RX}PL_{like}. \quad (17)$$

Similarly for a magnetic receive antenna

$$W_H = \frac{P_{RX,H}}{A_{nf,RX}} = P_{TX,E}PL_{unlike} \Rightarrow \frac{P_{RX,H}}{P_{TX,E}} = A_{nf,RX}PL_{unlike}. \quad (18)$$

It is important to realize that $A_{nf,RX}$ is not the effective area in the traditional far-field sense because the far-field effective area assumes a plane wave front. This effective area must also account for both the near- and far-field components (i.e., not just the θ- and ϕ- but also the r-component that is ignored in far-field analyses) and will therefore necessarily be different in almost all cases.

B. *Magnetic Transmit Antenna*

Using the principle of duality, the electric and magnetic fields of an infinitesimal current loop can be written as

$$\vec{E} = \frac{I\Delta S k^3}{4\pi\varepsilon_0 c}\left(\left(\frac{1}{kr}-\frac{j}{(kr)^2}\right)\sin(\theta)\hat{\phi}\right)e^{-jkr}, \text{ and} \quad (19)$$

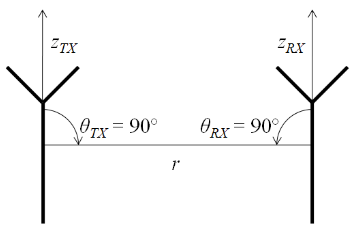

Fig. 2. Transmit and receive antenna with both at $\theta = 90°$.

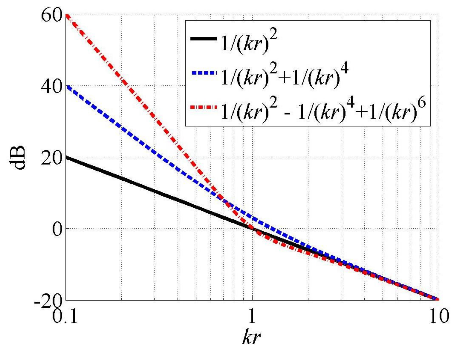

Fig. 3. Dependence on the near-field terms of the propagation formula.

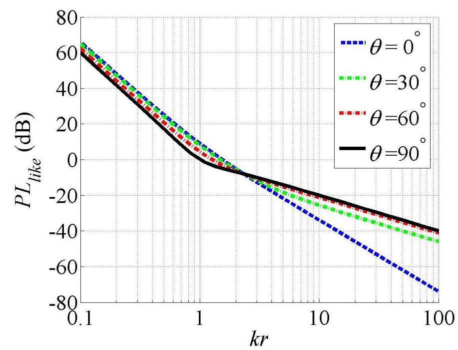

Fig. 4. Path-loss function for different θ.

$$\vec{H} = \frac{I\Delta S k^3}{4\pi}\left(\left(-\frac{1}{kr}+\frac{j}{(kr)^2}+\frac{1}{(kr)^3}\right)\sin(\theta)\hat{\theta} + 2\left(\frac{j}{(kr)^2}+\frac{1}{(kr)^3}\right)\cos(\theta)\hat{r}\right)e^{-jkr} . \quad (20)$$

where ΔS is the surface area of the loop. Using the same procedure outlined above for the electric transmitter, the following can be proven for a magnetic transmit antenna

$$\frac{P_{RX,H}}{P_{TX,H}} = A_{nf,RX} PL_{like}, \text{ and } \frac{P_{RX,E}}{P_{TX,H}} = A_{nf,RX} PL_{unlike} . \quad (21)$$

Thus, the near-field propagation formula can be summarized as below

$$\frac{P_{RX}(r,\theta,\phi)}{P_{TX}} = \begin{cases} A_{nf,RX} PL_{like}(r,\theta,\phi), \text{like antennas} \\ A_{nf,RX} PL_{unlike}(r,\theta,\phi), \text{unlike antennas} \end{cases}, \quad (22)$$

where "like antennas" are taken to mean either two electric or two magnetic antennas and "unlike antennas" are taken to mean one electric and one magnetic antenna.

IV. Comparing the Near-Field Propagation Formula with Friis's Formula

A. *The Special Case of $\theta = 90°$*

For the special case where both antennas are at $\theta = 90°$ ($z = 0$, the horizontal plane) relative to each other (as in Fig. 2), see in (5) and (6) that the electric and magnetic fields have no radial components. Therefore, the near-field effective areas of the antennas are equivalent to their far-field effective areas. For an infinitesimal dipole, this is

$$A_{nf,\inf} = A_{nf,TX} = A_{TX} = \frac{3\lambda^2}{8\pi} = \frac{3\pi}{2k^2}, \quad (23)$$

which further reduces (24) to:

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=90°,like} = \frac{k^4}{4\pi^2} A_{RX} A_{TX}\left(\frac{1}{(kr)^2}-\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right), \text{ and} \quad (24)$$

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=90°,unlike} = \frac{k^4}{4\pi^2} A_{RX} A_{TX}\left(\frac{1}{(kr)^2}+\frac{1}{(kr)^4}\right). \quad (25)$$

For large r, the $1/r^4$ and $1/r^6$ terms will go to zero much faster than the $1/r^2$ terms, and they can be ignored for all practical purposes, as demonstrated in Fig. 3. This leaves

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=90°,like} = \left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=90°,unlike} = \left(\frac{k}{2\pi r}\right)^2 A_{RX} A_{TX} = \frac{A_{RX} A_{TX}}{(\lambda r)^2} . \quad (26)$$

Thus, the near-field propagation formula converges to Friis's formula in the far field in the horizontal plane.

B. *Like Antenna Path-loss function and the Special Case of $\theta_{TX} = 0°$*

Fig. 4 shows a plot of PL_{like} as a function of kr for different values of θ_{TX}. The solid black line represents the special case of $\theta_{TX} = 90°$ that was previously demonstrated to follow the Friis formula path-loss function for large r. As θ_{TX} goes to 0 in the far field, the path-loss function gets smaller but still follows the $\theta_{TX} = 90°$ line. This is what one would expect from an ideal dipole with a donut power pattern in the far field. Finally, at $\theta_{TX} = 0°$, the path-loss function is smallest. The $\theta_{TX} = 0°$ line does not follow the $\theta_{TX} = 90°$ line because in the far field, that corresponds to the null of the antenna, where there is ideally no power.

However, the near field tells a different story. The path-loss function is largest for small r when $\theta_{TX} = 0°$, which in turn maximizes the P_{RX}/P_{TX} ratio. For $\theta_{TX} = 0°$, (22) reduces to

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=0°} = \begin{cases} \frac{3k^2}{2\pi} A_{nf,RX}\left(\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right), \text{like} \\ 0, \text{unlike} \end{cases} . \quad (27)$$

Since for $\theta_{TX} = 0°$ the only component present in the field equations of an ideal dipole is the radial component, it is convenient to define a near-field radial component pattern function of an ideal dipole as

$$F_{nf,r}(\theta,\phi) = \cos^2(\theta) . \quad (28)$$

The near-field radial component directivity can also be calculated

$$D_{nf,r}(\theta,\phi) = 4\pi \frac{F_{nf,r}(\theta,\phi)}{\int_0^{2\pi}\int_0^{\pi} F_{nf,r}(\theta,\phi)\sin(\theta)d\theta d\phi} = 4\pi \frac{\cos^2(\theta)}{\int_0^{2\pi}\int_0^{\pi} \cos^2(\theta)\sin(\theta)d\theta d\phi} = 3\cos^2(\theta) . \quad (29)$$

This function is maximum when $\theta_{RX} = 0°$, where the directivity is 3. Because an ideal dipole was assumed, the

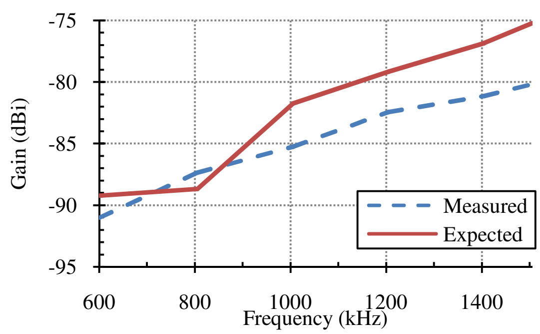

Fig. 5. Model LP-105 measured gain and expected gain.

directivity is equal to the gain. Therefore, the P_{RX}/P_{TX} ratio for the $\theta_{TX} = \theta_{RX} = 0°$ case can be further defined for the case of a general transmitter as

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=0^\circ} = \begin{cases} \dfrac{G_{nf,RX}G_{nf,TX}}{2}\left(\dfrac{1}{(kr)^4}+\dfrac{1}{(kr)^6}\right), \text{like} \\ 0, \text{unlike} \end{cases}. \tag{30}$$

Note that if a non-ideal dipole is assumed, the directivity is not equal to the gain, but the power transmitted in (14) will also be multiplied by the antenna efficiency. An analysis accounting for the antenna efficiency will discover the same result of (30).

V. Measuring Far-Field Gain of Electrically Small Antennas in the Near Field

In the case of θ = 90°, the effective areas in the near-field propagation formula are equivalent to the far-field effective areas. Therefore, the far-field gain can be measured in the near field for this special orientation. Equations (24) and (25) can also be written in terms of gain as

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=90^\circ,like} = \frac{G_{RX}G_{TX}}{4}\left(\frac{1}{(kr)^2}-\frac{1}{(kr)^4}+\frac{1}{(kr)^6}\right). \tag{31}$$

Assuming that the two antennas are of the same type, (31) can be used to measure gain. Three cases are discussed below.

A. Two Identical Antennas

If the antennas are the same, their gains should be the same. Therefore, by solving (31) for $G_{RX}G_{TX} = G^2$

$$G^2 = \frac{4P_{RX}}{P_{TX}\left(\dfrac{1}{(kr)^2}-\dfrac{1}{(kr)^4}+\dfrac{1}{(kr)^6}\right)} \;. \tag{32}$$
$$\Rightarrow G = 2(kr)^4\sqrt{\frac{P_{RX}}{P_{TX}((kr)^6-(kr)^4+(kr)^2)}}$$

We used this method to calculate the gain of two Empire (Singer) Model LP-105 loop antennas with known antenna factors. The measured results compared to the expected results are shown in Fig. 5. Sources of error in the measurement could include RF coupling through the power (coupling was noticeably apparent for electric antennas) and a non free space environment.

Note in the far-field limit as r gets large, the $(kr)^6$ will dominate the denominator, and (32) will reduce to

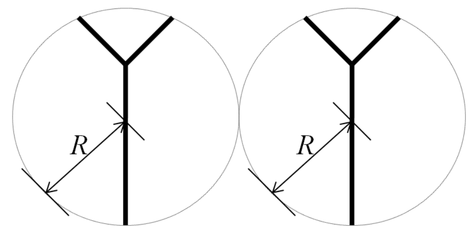

Fig. 6. Boundary spheres around two arbitrary antennas.

$$G = 2kr\sqrt{\frac{P_{RX}}{P_{TX}}}\;, \tag{33}$$

which is what Friis's formula also predicts.

B. One Antenna with Unknown Gain

If two like antennas are tested and the gain of one is known, the power transmitted and received can be measured and also substituted into (32). This results in

$$G_{unknown} = \frac{4P_{RX}}{P_{TX}G_{known}\left(\dfrac{1}{(kr)^2}-\dfrac{1}{(kr)^4}+\dfrac{1}{(kr)^6}\right)} \;. \tag{34}$$
$$= \frac{4P_{RX}(\beta r)^8}{P_{TX}G_{known}((kr)^6-(kr)^4+(kr)^2)}$$

Again, in the far-field limit as r gets large, the $(kr)^6$ will dominate the denominator and (34) will reduce to

$$G_{unknown} = (2kr)^2\frac{P_{RX}}{P_{TX}G_{known}}\;, \tag{35}$$

which again is what Friis's formula predicts.

C. Three Like Antennas

For three antennas of the same type (electric or magnetic), (34) can be used in the same method that Friis's formula is used in the far field to measure antenna gain of three unknown antennas [8]. Measure the power transmitted by each antenna and the power received by each of the other two antennas. This produces three simultaneous equations with three unknowns (the three gains) that can be solved.

VI. Fundamental Limit on Antenna Gain as a Function of Antenna Size

To derive the fundamental limit of antenna gain, first the concept of boundary spheres must be explained. They were originally introduced by Wheeler [9]. One of us [10] has applied them to the question of the maximum gain a given antenna can realize. However, that analysis considered only the θ = 90° case; since the maximum P_{RX}/P_{TX} ratio occurs for θ = 0°, the limit presented in [10] requires revision.

Imagine placing a boundary sphere around an arbitrary antenna with a radius R that is the smallest distance to completely enclose the antenna as shown in Fig. 6. Next, imagine a second identical antenna next to the first one also surrounded by a boundary sphere. Absent any other sources, the power received by one antenna cannot exceed the power transmitted by the other in order to comply with the law of conservation of energy. Mathematically,

$$\frac{P_{RX}}{P_{TX}} \le 1 . \quad (36)$$

Furthermore, the minimum separation between the antennas such that neither boundary sphere intersects is

$$r = 2R . \quad (37)$$

Take the maximum P_{RX}/P_{TX} ratio as (30). Combining (30), (36), and (37) results in:

$$\left.\frac{P_{RX}}{P_{TX}}\right|_{\theta=0^\circ} = \frac{G_{nf,RX} G_{nf,TX}}{2}\left(\frac{1}{(2kR)^4} + \frac{1}{(2kR)^6}\right) \le 1$$

$$\Rightarrow G_{nf,RX} G_{nf,TX} \le \frac{2}{\left(\frac{1}{(2kR)^4} + \frac{1}{(2kR)^6}\right)} = \frac{2(2kR)^6}{1+(2kR)^2} . \quad (38)$$

Because both antennas are assumed to be identical, their gains are also identical, so

$$G_{nf,RX} G_{nf,TX} = G_{nf}^{\ 2}$$

$$\Rightarrow G_{nf} \le \sqrt{\frac{2(2kR)^6}{1+(2kR)^2}} = (4\pi R_\lambda)^3 \sqrt{\frac{2}{1+(4\pi R_\lambda)^2}} , \quad (39)$$

where R_λ is taken to mean R in units of wavelength (so R/λ).

Finally, note that the near-field gain accounts for all components of the fields, whereas the far-field gain only accounts for two components. Therefore, one would expect that the near-field gain must be at least greater than or equal to the far-field gain. Therefore, in terms of the far-field gain

$$G_{ff} \le G_{nf} \le \sqrt{\frac{2(2kR)^6}{1+(2kR)^2}} = (4\pi R_\lambda)^3 \sqrt{\frac{2}{1+(4\pi R_\lambda)^2}} . \quad (40)$$

To check the limit suggested above, we measured the gains of a number of different magnetic antennas and compared them against their theoretical limit for their size (see Fig. 7). For the antennas with data taken at multiple frequencies, we measured the power received by an Empire LP-105 loop antenna with a known gain and changed the transmit antenna. We measured the gain of EMCO Model 6509 antenna using this method and compared it to its expected values based on its antenna factor. For the antennas with data at a single frequency, we first measured the power transmitted out to a known distance by the EMCO Model 6509 loop antenna with a known gain. We then transmitted the exact same power for each antenna at the same distance, and the relative difference of the power received by this new antenna was added to the gain of the EMCO.

Some antennas seem to perform better than the theoretical limit would predict. However, sources of error in the measurements, including RF coupling and non-free-space conditions, can account for this discrepancy. Recall that the Empire LP-105 antenna's measured gain was as high as 5 dB off of the expected value. Accounting for a measurement error of 5 dB, it is entirely plausible that all of the data fall below the expected limit.

VII. Conclusion

The near fields are an often overlooked aspect of antenna analysis that can often yield interesting results. For example, the gain limit derived above has profound implications on electrically small antennas, which are becoming more and more common as near-field applications are increasing in popularity. However, we suspect that we are only beginning to scratch the surface on this fascinating topic.

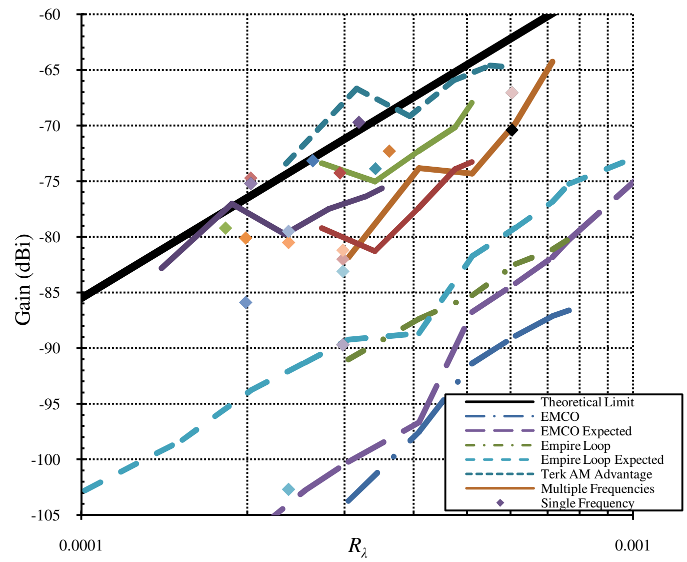

Fig. 7. Measured antenna gains compared to the theoretical limit.

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